

In Pursuit of Gamma¹

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ABSTRACT

After reviewing techniques for extracting clean information on CP-violating phase angles from B decays, we explain the rules for finding decay modes that can probe the phase angle γ of the unitarity triangle. We identify the more promising of these “ γ modes,” estimate their branching ratios, and examine the degree to which they are theoretically clean. We then show that when the quark mixing matrix is not approximated as usual, but is treated exactly, none of the “ γ modes” actually measures γ . Rather, each of them measures γ plus some correction. In all modes, the correction is small enough to be disregarded in first-generation experiments, but in some of them, it may be large enough to be observed in second-generation experiments.

Our treatment of the γ modes calls attention to the fact that when the quark mixing matrix is treated exactly, there are six unitarity triangles, rather than just one triangle. However, only four of the angles in these six triangles are independent. Examining the role played by these four angles, we discover that, in principle at least, measurements of nothing but CP-violating asymmetries in B decays are sufficient to determine the entire quark mixing matrix.

1. INTRODUCTION

In the Standard Model, CP violation is caused by complex elements in the unitary C(abibbo)-K(obayashi)-M(askawa) quark mixing matrix, V . If this Model is correct, then in many B decays there should be large CP-violating asymmetries from which theoretically clean information on the phases in V can be extracted. This information can then be used to confirm in detail that phases in V really are the origin of CP violation, or to exhibit an inconsistency of the theory.

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Figure 1: The unitarity triangle expressing orthogonality of the d and b columns of the CKM matrix.

In the Wolfenstein approximation¹ to V , there is a phase convention in which this matrix is real except for the elements V_{ub} and V_{td} . In this approximation, experiments on CP violation in B decays are usually described as probes of the angles α , β and γ of the “unitarity triangle,” shown in Fig. 1. That the legs in this Figure form a closed triangle follows from the unitarity constraint that the d and b columns of V must be orthogonal, and the assumption that there are only three generations. From Fig. 1 we see that in the Wolfenstein approximation,

$$\gamma \simeq -\arg(V_{ub}) , \quad \beta \simeq -\arg(V_{td}) , \quad \alpha \simeq \pi + \arg(V_{ub}) + \arg(V_{td}). \quad (1)$$

Thus, probing the angle γ – the focus of the Gamma Working Group within this Snowmass Workshop – amounts to probing the phase of V_{ub} .

In Sec. 2, we recall what must be measured to extract clean CKM phase information from the decays of *neutral* B mesons. We then identify a number of B decay modes which potentially can yield the phase of V_{ub} . We estimate the branching ratios, and comment on the degree of theoretical cleanliness, of these modes.

In Sec. 3, we ask what happens if one treats the CKM matrix exactly, rather than in the Wolfenstein approximation. The single unitarity triangle of Fig. 1 is then replaced by the six unitarity triangles of Fig. 9, which express the orthogonality of any pair of columns, or any pair of rows, of V . We find that when the Wolfenstein approximation is not made, *none* of the B decays modes proposed so far as “probes of the angle γ ” actually measure γ . Instead, each of these modes measures γ plus corrections which are small angles in the triangles other than the one of Fig. 1. We explore the degree to which these corrections may undermine the interpretation of these decay modes as probes of γ .

In Sec. 4, we report on a general analysis of the unitarity triangles and CP-violating phases when the CKM matrix is treated exactly. We find that the CKM phase yielded by a theoretically-clean decay mode is always a simple linear combination of angles in the unitarity triangles. Moreover, at least in principle, measurements of CP-violating asymmetries in B decays are sufficient to determine, not only some angles in one unitarity triangle, but the entire CKM matrix.

2. POTENTIAL PROBES OF GAMMA

2.1 Extraction of CKM Phases

In general, *theoretically clean* CKM phase information can be extracted only from the decays of the *neutral* B mesons, B_d and B_s . In the decays of either of these mesons, we are usually interested in some final state which can come both from the pure B and from the pure \bar{B} . Now, owing to B - \bar{B} mixing, a particle born at time $t = 0$ as a pure $|B_q\rangle$, $q = d$ or s , evolves in time t into a state $|B_q(t)\rangle$ which is a linear superposition of $|B_q\rangle$ and $|\bar{B}_q\rangle$.^{2,3} In the (excellent) approximation that B - \bar{B} mixing is dominated by a t -quark box diagram, this superposition is given by

$$|B_q(t)\rangle = \exp\left(-i(m_q - i\frac{\Gamma_q}{2})t\right) [c_q|B_q\rangle + i\omega_q s_q|\bar{B}_q\rangle]. \quad (2)$$

Here, m_q is the average mass of the two mass eigenstates of the B_q - \bar{B}_q system, and Γ_q is their common width.⁴ With Δm_q their mass difference,

$$c_q \equiv \cos\left(\frac{\Delta m_q}{2}t\right), \text{ and } s_q \equiv \sin\left(\frac{\Delta m_q}{2}t\right). \quad (3)$$

Finally, ω_q is the CKM phase of the amplitude $A(B_q \rightarrow \bar{B}_q)$ for $|B_q\rangle \rightarrow |\bar{B}_q\rangle$, and is given by

$$\omega_q = \frac{V_{tq}V_{tb}^*}{V_{tq}^*V_{tb}}. \quad (4)$$

In a similar fashion, a particle born at $t = 0$ as a pure $|\bar{B}_q\rangle$ evolves in time t into a linear superposition $|\bar{B}_q(t)\rangle$ of $|\bar{B}_q\rangle$ and $|B_q\rangle$ given by an expression analogous to that of Eq. (2). Suppose, now, that f is some final state which can come both from a pure B_q and from a pure \bar{B}_q . From Eq. (2), the amplitude $A(B_q(t) \rightarrow f)$ for the meson $B_q(t)$ which at time $t = 0$ was a pure B_q to decay into f at time t is

$$A(B_q(t) \rightarrow f) = \exp\left(-i(m_q - i\frac{\Gamma_q}{2})t\right) [c_q A(B_q \rightarrow f) + i\omega_q s_q A(\bar{B}_q \rightarrow f)]. \quad (5)$$

The corresponding time-dependent decay rate, $\Gamma_{q,f}(t) \equiv |A(B_q(t) \rightarrow f)|^2$, then contains a term representing the interference between the $A(B_q \rightarrow f)$ and $A(\bar{B}_q \rightarrow f)$ terms in Eq. (5).

Let us now turn to the CP-mirror-image process $\bar{B}_q(t) \rightarrow \bar{f}$, in which the meson $\bar{B}_q(t)$ born at $t = 0$ as a pure \bar{B}_q decays into the final state \bar{f} , the CP-mirror-image of f . The rate for this process, $\bar{\Gamma}_{q,\bar{f}}(t) \equiv |A(\bar{B}_q(t) \rightarrow \bar{f})|^2$, also contains an $A(B_q \rightarrow f)$ - $A(\bar{B}_q \rightarrow f)$ interference term. However, when the CKM matrix elements are complex, this interference term has, in general, a different magnitude than its counterpart in $\Gamma_{q,f}(t)$. This difference leads to a CP-violating difference between $\bar{\Gamma}_{q,\bar{f}}(t)$ and $\Gamma_{q,f}(t)$, which one would like to observe. In order to observe it, one must know in each event whether the decaying meson was born as a B_q or a \bar{B}_q . That is, one must tag it as one of these by observing a flavor-revealing decay of an accompanying beautiful meson or baryon. It may also be possible to use an interesting, recently-proposed “self-tagging” method.⁵

Suppose that the final state f is a CP eigenstate, so that \bar{f} is the same as f . If f has intrinsic CP parity η_f , the decay rates $\Gamma_{q,f}(t)$ and $\bar{\Gamma}_{q,\bar{f}}(t) = \bar{\Gamma}_{q,f}(t)$ are given by²

$$\begin{aligned} \Gamma_{q,f}(t) &\propto \exp(-\Gamma_q t) [1 + \eta_f \sin \varphi_{q,f} \sin(\Delta m_q t)], \\ \bar{\Gamma}_{q,f}(t) &\propto \exp(-\Gamma_q t) [1 - \eta_f \sin \varphi_{q,f} \sin(\Delta m_q t)]. \end{aligned} \quad (6)$$

Here, $\varphi_{q,f}$ is the phase of some product of CKM elements whose identities depend on q and f . It is $\varphi_{q,f}$ that we would like to determine from the asymmetry in the decay rates (6). We shall be interested in decays where $\varphi_{q,f}$ is γ , or perhaps 2γ .

Suppose, next, that the final state f is not a CP eigenstate, but has a CP conjugate \bar{f} distinct from itself. An example of interest is $f = D_s^+ K^-$, $\bar{f} = D_s^- K^+$. Theoretically clean CKM phase information can still be extracted.⁶ There are now four decay rates which can be measured. They are given by

$$\begin{aligned}\Gamma_{q,f}(t) &= \exp(-\Gamma_q t) \left[c_q^2 M_{q,f}^2 + s_q^2 \bar{M}_{q,f}^2 + 2c_q s_q M_{q,f} \bar{M}_{q,f} \sin(\varphi_{q,f} + \theta_{q,f}) \right] \\ \bar{\Gamma}_{q,\bar{f}}(t) &= \exp(-\Gamma_q t) \left[c_q^2 M_{q,f}^2 + s_q^2 \bar{M}_{q,f}^2 + 2c_q s_q M_{q,f} \bar{M}_{q,f} \sin(-\varphi_{q,f} + \theta_{q,f}) \right] \\ \bar{\Gamma}_{q,f}(t) &= \exp(-\Gamma_q t) \left[c_q^2 \bar{M}_{q,f}^2 + s_q^2 M_{q,f}^2 - 2c_q s_q M_{q,f} \bar{M}_{q,f} \sin(\varphi_{q,f} + \theta_{q,f}) \right] \\ \Gamma_{q,\bar{f}}(t) &= \exp(-\Gamma_q t) \left[c_q^2 \bar{M}_{q,f}^2 + s_q^2 M_{q,f}^2 - 2c_q s_q M_{q,f} \bar{M}_{q,f} \sin(-\varphi_{q,f} + \theta_{q,f}) \right].\end{aligned}\quad (7)$$

Here, $\Gamma_{q,f}(t)$ is the rate for decay of $B_q(t)$ into f , $\bar{\Gamma}_{q,\bar{f}}(t)$ is the rate for decay of $\bar{B}_q(t)$ into \bar{f} , etc. The angle $\varphi_{q,f}$ is, as before, the phase of some product of CKM elements whose identities depend on q and f . As before, $\varphi_{q,f}$ is the quantity we would like to determine, and we shall be interested here in decays where $\varphi_{q,f}$ is γ or 2γ . The constants $M_{q,f}$ and $\bar{M}_{q,f}$ are, respectively, the magnitudes of the amplitudes $A(B_q \rightarrow f)$ and $A(\bar{B}_q \rightarrow f)$. It is desirable that $M_{q,f}$ and $\bar{M}_{q,f}$ be comparable, so that the rates (7) will be sensitive to $\varphi_{q,f}$. Finally, $\theta_{q,f}$ is a strong-interaction phase.

With Γ_q and Δm_q known, measuring the decay rates (7) more than suffices to determine the quantities $s_{\pm}(q, f) \equiv \sin(\pm\varphi_{q,f} + \theta_{q,f})$. In turn, these quantities determine $\sin^2 \varphi_{q,f}$, up to a two-fold ambiguity, via the expression

$$\sin^2 \varphi_{q,f} = \frac{1}{2} \left[1 - s_+(q, f)s_-(q, f) \pm \sqrt{(1 - s_+^2(q, f))(1 - s_-^2(q, f))} \right]. \quad (8)$$

If, contrary to what we have assumed here, the two mass eigenstates of the B_q - \bar{B}_q system have widths which differ enough to result in measurable effects, it becomes possible to experimentally resolve some of the ambiguities in the determination of $\varphi_{q,f}$.

The decay rates (6) and (7) hold when $A(B_q \rightarrow f)$ and $A(\bar{B}_q \rightarrow f)$ are each dominated by a single Feynman diagram, so that they each have a well-defined CKM phase. When, instead, $A(B_q \rightarrow f)$ receives significant contributions from several Feynman diagrams with different CKM phases, the extraction of clean CKM phase information from experimental decay rates is either impossible or requires measurement of rates for several isospin-related decays.⁷ When several diagrams contribute significantly, the largest one is usually a tree diagram, and the others are usually penguin diagrams. In exploring the usefulness of each decay mode proposed as a probe of the angle γ , we will consider the degree to which penguin or other diagrams with CKM phases different from that of the dominant diagram might contribute significantly to the mode.

In the decay rates (6) and (7), the violation of CP invariance, and the information on the CKM phase $\varphi_{q,f}$ producing this violation, are in the term proportional to $2c_q s_q = \sin(\Delta m_q t)$. To learn about $\varphi_{q,f}$, one would like to measure the time dependence of the rates and uncover this term. When f is not a CP eigenstate and the rates are described by Eqs. (7), measurement of their time dependence is absolutely essential. Not all four decay rates need be measured. Indeed, it is easy to see³ that, say, $\Gamma_{q,f}(t)$ and $\bar{\Gamma}_{q,\bar{f}}(t)$ alone suffice to determine $\sin^2 \varphi_{q,f}$. However, if we measure only the time integrals of the rates (7), then

we cannot determine $\sin^2 \varphi_{q,f}$, even if we measure the time integrals of all four of the rates. For, if Eqs. (7) hold, then clearly we must have

$$\Gamma_{q,f}(t) + \bar{\Gamma}_{q,f}(t) = \Gamma_{q,\bar{f}}(t) + \bar{\Gamma}_{q,\bar{f}}(t) . \quad (9)$$

Now, when the decay rates in this constraint are replaced by their time integrals, they become merely four numbers, instead of four functions of time, and the constraint implies that only *three* of these four numbers are independent. But the decay rates (7), and their time-integrated analogues, depend on *four* unknowns: $M_{q,f}$, $\bar{M}_{q,f}$, $\varphi_{q,f}$, and $\theta_{q,f}$. Hence, it is impossible to determine $\varphi_{q,f}$ from the time-integrated rates. When f is a CP eigenstate and the decay rates are described by Eqs. (6), then in principle one can extract $\sin \varphi_{q,f}$ from a knowledge of the time-integrated rates alone. However, in the case of B_s decay, this will be extremely difficult if, as we expect,⁸ Δm_s is an order of magnitude larger than Γ_s . When $x_s \equiv \Delta m_s/\Gamma_s$ is large, the fractional contribution of the CP-violating $\sin \varphi_{q,f} \sin(\Delta m_s t)$ term in Eqs. (6) to the decay rate gets reduced by a factor of $\sim 1/x_s$ when the rate is integrated over time.

In view of these circumstances, we assume here that when one is seeking to extract CKM phase information from a neutral B decay, the time dependence of the decay rate must be measured, except perhaps in B_d decay to a CP eigenstate.

2.2 Neutral B Decay Modes That Can Probe Gamma

In which neutral B decays can we identify the CKM phase $\varphi_{q,f}$ that is probed as γ or 2γ ? As we have discussed, the CP violation that we study in the decay $B_q(t) \rightarrow f$ results from interference between the two terms in the decay amplitude (5). The CKM phase $\varphi_{q,f}$ that is probed by $B_q(t) \rightarrow f$ is, therefore, just the relative CKM phase of these two terms. Thus, remembering that ω_q is the CKM phase of $A(B_q \rightarrow \bar{B}_q)$, we see from Eq. (5) that the $\varphi_{q,f}$ probed by $B_q(t) \rightarrow f$ is given by

$$\varphi_{q,f} = \text{CKM Phase} \left[\frac{A(B_q \rightarrow f)}{A(B_q \rightarrow \bar{B}_q) \times A(\bar{B}_q \rightarrow f)} \right] . \quad (10)$$

Instead of referring to Eq. (5), we may think of the CP violation in $B_q(t) \rightarrow f$ as resulting from interference between the amplitude $A(B_q \rightarrow f)$ for the particle born as a pure B_q to decay directly to f , and the amplitude $A(B_q \rightarrow \bar{B}_q) \times A(\bar{B}_q \rightarrow f)$ for this particle to convert, via mixing, into a \bar{B}_q which then decays into f . Once again we conclude that the $\varphi_{q,f}$ probed by $B_q(t) \rightarrow f$ is given by (10).

Now, recall that in the Wolfenstein approximation to the CKM matrix, all CKM elements are real save V_{ub} and V_{td} , and $\gamma = -\arg(V_{ub})$. In this approximation, the CKM phase of $A(B_d \rightarrow \bar{B}_d)$ is

$$\arg(V_{td}/V_{td}^*) = -2\beta , \quad (11)$$

while that of $A(B_s \rightarrow \bar{B}_s)$ is

$$\arg(V_{ts}/V_{ts}^*) = 0 . \quad (12)$$

Thus, from Eq. (10), we can probe γ by studying $B_s(t)$ decays in which the phase of $A(B_s \rightarrow f)/A(\bar{B}_s \rightarrow f)$ is essentially γ . This will be the case when each of $B_s \rightarrow f$ and $\bar{B}_s \rightarrow f$ is dominated by a tree diagram, and either (a) the tree diagram for $B_s \rightarrow f$ involves

Decay Mode	Branching Ratio
$B_s \rightarrow D_s^\pm K^\mp$	2×10^{-4}
$B_s \rightarrow D^0 \phi, \overline{D^0} \phi$	2×10^{-5}
$B_s \rightarrow \rho^0 K_S$	5×10^{-7}

Table 1: B_s decay modes that can probe the angle γ .

one of the processes

$$\bar{b} \rightarrow \bar{u} + \begin{cases} c\bar{s} \\ c\bar{d} \\ u\bar{s} \\ u\bar{d} \end{cases}, \quad (13)$$

or (b) the tree diagram for $\overline{B}_s \rightarrow f$ involves one of the processes

$$b \rightarrow u + \begin{cases} \bar{c}s \\ \bar{c}d \\ \bar{u}s \\ \bar{u}d \end{cases}, \quad (14)$$

or both. When both (13) and (14) are involved, the CKM phase of $A(B_s \rightarrow f)/A(\overline{B}_s \rightarrow f)$ is obviously $\arg(V_{ub}^*/V_{ub}) = 2\gamma$. When only one of them is involved, the other is replaced by a (real) $b \rightarrow c$ or $\bar{b} \rightarrow \bar{c}$ transition, so that the phase of $A(B_s \rightarrow f)/A(\overline{B}_s \rightarrow f)$ is γ .

We have considered the hadronic B_s decay modes produced by tree diagrams for the quark processes (13,14). We have tried to identify the modes that have advantageous branching ratios, and in which the interfering decay amplitudes $A(B_s \rightarrow f)$ and $A(\overline{B}_s \rightarrow f)$ are each dominated by a single Feynman diagram⁹ and have comparable magnitudes. The most promising modes we found are listed, together with their estimated branching ratios, in Table 1. These branching ratios were obtained by comparing the modes of interest to others whose branching ratios are already known. We now discuss the modes in Table 1 in turn.

- $B_s(t), \overline{B}_s(t) \rightarrow D_s^\pm K^\mp$:¹⁰

Here the final state $f \equiv D_s^+ K^-$ is distinct from its CP conjugate, $\bar{f} \equiv D_s^- K^+$, and one uses the expressions (7) to analyze the time-dependent decays of $B_s(t)$ and $\overline{B}_s(t)$ into f and \bar{f} . Tagging of the parent meson and study of the time dependence of the decays are essential.

The expected branching ratio is relatively large. The value quoted in Table 1, 2×10^{-4} , is for the decay $B_s \rightarrow D_s^- K^+$. Like all the values quoted, it assumes the parent to be a pure B_s and neglects mixing. The value 2×10^{-4} , which should be fairly reliable, is obtained by comparing the dominant diagram for $B_s \rightarrow D_s^- K^+$, shown in Fig. 2, to the very similar one for the decay $B_d \rightarrow D^- \pi^+$, whose branching ratio is known. The branching ratio for the decay $B_s \rightarrow D_s^+ K^-$ (again of a pure B_s neglecting mixing) is estimated, both in Ref. 10 and by the present authors, to be $\sim 1 \times 10^{-4}$. This estimate is obtained by comparing the dominant diagram for $B_s \rightarrow D_s^+ K^-$, shown in Fig. 3, to the related but somewhat different diagram for $B^- \rightarrow \Psi K^-$. Accordingly, it is not as reliable as the estimate for $B_s \rightarrow D_s^- K^+$.

In $B_s(t) \rightarrow D_s^+ K^-$, the interfering decay amplitudes are, of course, $A(B_s \rightarrow D_s^+ K^-)$ and $A(\overline{B}_s \rightarrow D_s^+ K^-)$. The amplitude $A(B_s \rightarrow D_s^+ K^-)$, being dominated by the diagram of

Figure 2: The dominant diagram for $B_s \rightarrow D_s^- K^+$.

Figure 3: The dominant diagram for $B_s \rightarrow D_s^+ K^-$.

Fig. 3, has a CKM phase which is $\arg(V_{ub}^* V_{cs}) \simeq \gamma$. This amplitude receives no other tree-level contributions except from a W -exchange diagram with the same CKM phase. Penguin diagrams cannot contribute at all. The amplitude $A(\overline{B}_s \rightarrow D_s^+ K^-)$ is dominated by the diagram which is the CP-mirror-image of that in Fig. 2. Thus, it has a CKM phase which is $\arg(V_{cb} V_{us}^*) \simeq 0$. It receives no other tree-level contributions except from a W -exchange diagram with the same CKM phase, and no penguin contributions. Thus, from Eq. (10), the CKM phase $\varphi_{s,D_s^+ K^-}$ probed by the rates (7) for the decays $B_s(t), \overline{B}_s(t) \rightarrow D_s^\pm K^\mp$ is γ . Moreover, from our branching ratio estimates, the magnitudes $M_{s,D_s^+ K^-} = |A(B_s \rightarrow D_s^+ K^-)|$ and $\overline{M}_{s,D_s^+ K^-} = |A(\overline{B}_s \rightarrow D_s^+ K^-)|$ ($= |A(B_s \rightarrow D_s^- K^+)|$) of the two interfering decay amplitudes in $B_s(t) \rightarrow D_s^+ K^-$ are in the ratio $(1 \times 10^{-4}/2 \times 10^{-4})^{1/2} \simeq 0.7$. Thus, the desire that these magnitudes be comparable is very nicely satisfied.

- $B_s(t), \overline{B}_s(t) \rightarrow D^0 \phi, \overline{D^0} \phi$:

Once again, we have a final state, $f = D^0 \phi$, which is distinct from its CP conjugate, $\bar{f} = \overline{D^0} \phi$, and we use the expressions (7) to analyze the four time-dependent decays $B_s(t) \rightarrow D^0 \phi$, $B_s(t) \rightarrow \overline{D^0} \phi$, $\overline{B}_s(t) \rightarrow D^0 \phi$, and $\overline{B}_s(t) \rightarrow \overline{D^0} \phi$. Tagging of the parent B is essential.

In $B_s(t) \rightarrow D^0 \phi$, the interfering decay processes are $B_s \rightarrow D^0 \phi$, which is dominated by the tree diagram in Fig. 4, and $\overline{B}_s \rightarrow D^0 \phi$, which is dominated by the tree diagram in Fig. 5. Penguin diagrams cannot contribute. Thus, the CKM phase $\varphi_{s,D^0 \phi}$ probed by $B_s(t) \rightarrow D^0 \phi$ and the related decays is γ .

The branching ratio estimate quoted in Table 1, 2×10^{-5} , is for $B_s \rightarrow \overline{D^0} \phi$, or for its CP-mirror-image $\overline{B}_s \rightarrow D^0 \phi$, and is obtained by comparing this process to $B_d \rightarrow \Psi K^{*0}$. The diagram which dominates $B_s \rightarrow D^0 \phi$ is almost identical to that which dominates $\overline{B}_s \rightarrow$

Figure 4: The dominant diagram for $B_s \rightarrow D^0 \phi$.

Figure 5: The dominant diagram for $\overline{B}_s \rightarrow D^0 \phi$.

$D^0 \phi$, apart from CKM factors, and we estimate the branching ratio for $B_s \rightarrow D^0 \phi$ to be 4×10^{-6} . The magnitudes $M_{s,D^0 \phi} = |A(B_s \rightarrow D^0 \phi)|$ and $\overline{M}_{s,D^0 \phi} = |A(\overline{B}_s \rightarrow D^0 \phi)|$ of the two interfering decay amplitudes in $B_s(t) \rightarrow D^0 \phi$ then have the ratio $(4 \times 10^{-6} / 2 \times 10^{-5})^{1/2} \simeq 0.4$, which is $O(1)$, as desired.¹¹

- $B_s(t), \overline{B}_s(t) \rightarrow \rho^0 K_S$:

This mode, oft-proposed as a probe of γ , has the advantage of yielding a CP eigenstate, so that the analysis is simplified. However, the estimated branching ratio, obtained by comparing $B_s \rightarrow \rho^0 K_S$ to $B_d \rightarrow \Psi K_S$, is very small.

The decay amplitudes $A(B_s \rightarrow \rho^0 K_S)$ and $A(\overline{B}_s \rightarrow \rho^0 K_S)$ that interfere in $B_s(t) \rightarrow \rho^0 K_S$ are dominated, respectively, by the tree diagram in Fig. 6 and by its CP-mirror-image. Thus, from Eq. (10), the CKM phase $\varphi_{s,\rho^0 K_S}$ probed by $B_s(t), \overline{B}_s(t) \rightarrow \rho^0 K_S$ via Eqs. (6) is 2γ . Furthermore, as in all decays to a CP eigenstate, if one diagram dominates $A(B_q \rightarrow f)$ and its CP-mirror-image dominates $A(\overline{B}_q \rightarrow f)$, these two interfering decay amplitudes are of identical size. However, unlike the other modes in Table 1, $B_s(t), \overline{B}_s(t) \rightarrow \rho^0 K_S$ does involve penguin contributions. Possibly, these are significant, and some of them have CKM phases other than γ . Thus, in addition to having a small branching ratio, $B_s(t), \overline{B}_s(t) \rightarrow \rho^0 K_S$ may not be a clean probe of γ .

2.3 Non- B_s Decay Modes That Can Probe Gamma

While most decays of charged B mesons cannot yield clean CKM phase information, the decays $B^\pm \rightarrow DK^\pm$ are an exception, and they probe γ .¹² The technique for using these decays, explained in Ref. 12, requires one to measure the branching ratios for $B^\pm \rightarrow D^0 K^\pm$,

Figure 6: The dominant diagram for $B_s \rightarrow \rho^0 K_s$.

Figure 7: The dominant diagram for $B^+ \rightarrow D^0 K^+$.

$B^\pm \rightarrow \overline{D^0} K^\pm$, and $B^\pm \rightarrow D_{CP} K^\pm$, where D_{CP} is a neutral D that decays to a CP eigenstate such as $K^+ K^-$ or $\pi^+ \pi^-$. As in all charged B decays, there is, of course, no need to tag, and no non-exponential time dependence.

The decay $B^+ \rightarrow D^0 K^+$ is dominated by the diagram in Fig. 7, while $B^+ \rightarrow \overline{D^0} K^+$ is dominated by the diagram in Fig. 8. Since D_{CP} is a coherent superposition of D^0 and $\overline{D^0}$, in $B^+ \rightarrow D_{CP} K^+$ the diagrams of Figs. 7 and 8 interfere. Now, the CKM phase of the $B^+ \rightarrow D^0 K^+$ diagram, Fig. 7, is $\arg(V_{ub}^* V_{cs}) \simeq \gamma$. That of the $B^+ \rightarrow \overline{D^0} K^+$ diagram, Fig. 8, is $\arg(V_{cb}^* V_{us}) \simeq 0$. Hence, the interference between these diagrams probes γ . There are no penguin contributions, so this probe is quite clean.

Figure 8: The dominant diagram for $B^+ \rightarrow \overline{D^0} K^+$.

By comparing the diagram for $B^+ \rightarrow \overline{D^0}K^+$ to that for $B^+ \rightarrow \overline{D^0}\pi^+$, we readily estimate that $BR(B^+ \rightarrow \overline{D^0}K^+) \simeq 2 \times 10^{-4}$. This is a promising value. However, by comparing the diagram for $B^+ \rightarrow D^0K^+$ to that for $B^+ \rightarrow \Psi K^+$, we estimate that $BR(B^+ \rightarrow D^0K^+) \sim 2 \times 10^{-6}$. In addition, by comparing $B^+ \rightarrow D^0K^+$ to $B_d \rightarrow \overline{D^0}\pi^0$, for which there is an interesting upper limit,¹³ we estimate that $BR(B^+ \rightarrow D^0K^+) \lesssim 6 \times 10^{-6}$. Thus, if these estimates prove to be right, this branching ratio may be hard to measure. So may the branching ratios for $B^\pm \rightarrow D_{CP}K^\pm$, since study of these processes requires that the neutral D decay to a CP eigenstate, a requirement which costs a factor of $\sim 10^{-2}$ in overall branching ratio. The initial B^\pm decay will be dominated by the diagram of Fig. 8 or its CP-mirror-image, and so will have a branching ratio of $\simeq 2 \times 10^{-4}$. Thus, the overall branching ratio will be $\sim 2 \times 10^{-6}$.

As in all studies of CP violation in B decay, one would like the two diagrams which interfere in $B^+ \rightarrow D_{CP}K^+$ to be of comparable magnitude. From our branching ratio estimates, their magnitudes will be in the ratio $\sim (2 \times 10^{-6}/2 \times 10^{-4})^{1/2} \simeq 1/10$. While not as close to unity as one might wish, this ratio is perhaps close enough to yield measurable interference effects.

A variant of the $B^\pm \rightarrow DK^\pm$ approach utilizing the self-tagging B_d decays $B_d(\overline{B}_d) \rightarrow DK^{*0}(\overline{D}\overline{K}^{*0})$ has been proposed as an alternate way to probe γ .¹⁴ By comparing to $B^+ \rightarrow D^0K^+$, we estimate that $BR(B_d \rightarrow D^0K^{*0}) \sim 2 \times 10^{-6}$, and by comparing to $B_d \rightarrow D^0K^{*0}$, that $BR(B_d \rightarrow \overline{D^0}K^{*0}) \sim 2 \times 10^{-5}$.

3. WHAT ANGLES DO THE “GAMMA” MODES ACTUALLY MEASURE?

We have identified a number of B decay modes which, *within the Wolfenstein approximation to the CKM matrix*, probe the angle γ . In each of these modes, the decay amplitude consists of two interfering terms, as illustrated in Eq. (5), and each of these terms is dominated by a single Feynman diagram. *In the Wolfenstein approximation*, the CKM phases of these dominating diagrams are such that the interfering terms in the decay amplitude have relative CKM phase γ , or 2γ . In this approximation, the statement that our “ γ modes” probe γ entails only the error, which we have argued is in most cases small or absent, corresponding to the neglect of the non-dominating diagrams. However, suppose that one does not make the Wolfenstein approximation. The CKM phases of Feynman diagrams are then altered. Neglecting the non-dominating diagrams, do the “ γ modes” still probe γ ? If not, what phase angle does each of them actually probe? How big an error do we make if we identify this angle as being approximately γ ?

To explore these questions, we note that a very useful framework for dealing with phases in the CKM matrix is provided by the “unitarity triangles.” One of these triangles is shown in Fig. 1. When the CKM matrix is treated exactly, rather than in the Wolfenstein approximation, one has, not just this one triangle, but six triangles. These triangles correspond to the unitarity constraint that any pair of columns, or any pair of rows, of the CKM matrix be orthogonal. That is, they correspond to the orthogonality requirements

$$\begin{array}{ll}
ds & V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \\
& \lambda \qquad \qquad \lambda \qquad \qquad \lambda^5 \\
sb & V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \\
& \lambda^4 \qquad \qquad \lambda^2 \qquad \qquad \lambda^2
\end{array}$$

$$\begin{aligned}
db & \quad \frac{V_{ud}V_{ub}^*}{\lambda^3} + \frac{V_{cd}V_{cb}^*}{\lambda^3} + \frac{V_{td}V_{tb}^*}{\lambda^3} = 0 \\
uc & \quad \frac{V_{ud}V_{cd}^*}{\lambda} + \frac{V_{us}V_{cs}^*}{\lambda} + \frac{V_{ub}V_{cb}^*}{\lambda^5} = 0 \\
ct & \quad \frac{V_{cd}V_{td}^*}{\lambda^4} + \frac{V_{cs}V_{ts}^*}{\lambda^2} + \frac{V_{cb}V_{tb}^*}{\lambda^2} = 0 \\
ut & \quad \frac{V_{ud}V_{td}^*}{\lambda^3} + \frac{V_{us}V_{ts}^*}{\lambda^3} + \frac{V_{ub}V_{tb}^*}{\lambda^3} = 0
\end{aligned} \tag{15}$$

To the left of each of these equations, we have indicated the pair of columns, or pair of rows, whose orthogonality it expresses. Also, under each term in the equations, we have indicated the rough magnitude of the term as a power of the Wolfenstein parameter $\lambda = 0.22$.

The unitarity triangles, depicted somewhat schematically in Fig. 9, are simply pictures in the complex plane of the conditions (15). Apart from signs and extra π 's, the angles in any triangle are just the relative phases of the various terms in the corresponding condition. Let us refer to a specific triangle by stating the columns (rows) whose orthogonality it expresses, and a specific leg in this triangle by stating which up-type (down-type) quark it involves. Denoting up-type quarks by Greek letters, and down-type ones by Latin letters, let

$$\omega_{\alpha\beta}^{ij} \equiv \arg \left(V_{\alpha i} V_{\alpha j}^* / V_{\beta i} V_{\beta j}^* \right) \tag{16}$$

be the relative phase of the α and β legs in the ij triangle. Since

$$\arg \left(V_{\alpha i} V_{\alpha j}^* / V_{\beta i} V_{\beta j}^* \right) = \arg \left(V_{\alpha i} V_{\beta i}^* / V_{\alpha j} V_{\beta j}^* \right), \tag{17}$$

$\omega_{\alpha\beta}^{ij}$ is also the relative phase of the i and j legs in the $\alpha\beta$ triangle. That is, each angle in a triangle expressing orthogonality of rows is also an angle in one expressing orthogonality of columns. Hence, for our discussion of CKM phases, we can forget about the row triangles. From the first two of Eqs. (15) (c.f. also Fig. 9), we see that

$$\begin{aligned}
\omega_{uc}^{ds} & \leq O(\lambda^4), \\
\omega_{ct}^{sb} & \leq O(\lambda^2).
\end{aligned} \tag{18}$$

That is, one of the angles in the sb triangle is small ($\lesssim 0.05$), and one in the ds triangle is extremely small ($\lesssim 0.003$). (There is no reason to suppose that the remaining angles in these triangles are small.)

Now, when the CKM matrix is treated exactly, what CKM phases do the decay modes we have discussed in Sec. 2 actually probe? Any neutral B mode probes the phase given by Eq. (10). Applied to any $B_s(t)$ decay, this equation involves the mixing phase $\arg(B_s \rightarrow \overline{B}_s)$, which from Eq. (4) is $\arg(V_{ts}V_{tb}^*/V_{ts}^*V_{tb})$. Moreover, the V_{ts}/V_{ts}^* in this expression cannot be cancelled by the decay amplitudes $A(B_s \rightarrow f)$ and $A(\overline{B}_s \rightarrow f)$ so long as these amplitudes are dominated by tree diagrams, which can never involve a t quark. However, from Fig. 1, apart from a π ,

$$\gamma \equiv \omega_{uc}^{db} = \arg(V_{ud}V_{ub}^*/V_{cd}V_{cb}^*). \tag{19}$$

Figure 9: The unitarity triangles. To the left of each triangle is indicated the pair of columns, or of rows, whose orthogonality this closed triangle expresses.

Since the CKM elements in this expression do not include V_{ts} , it is clear that the phase probed by a $B_s(t)$ decay cannot be γ .

For the decay $B_s(t) \rightarrow D_s^+ K^-$, $A(B_s \rightarrow f)$ is dominated by the diagram in Fig. 3, proportional to $V_{cs}V_{ub}^*$. Similarly, $A(\overline{B}_s \rightarrow f)$ is dominated by the CP-mirror-image of the diagram in Fig. 2, and so is proportional to $V_{cb}V_{us}^*$. Thus, from Eqs. (10) and (4), the phase probed by $B_s(t) \rightarrow D_s^+ K^-$ is

$$\begin{aligned}\varphi_{s,D_s^+ K^-} &= \arg \left[\frac{V_{cs}V_{ub}^*}{\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} V_{cb}V_{us}^*} \right] \\ &= \arg \left[\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \left(\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*} \right)^2 \frac{V_{us}V_{ud}^*}{V_{cs}V_{cd}^*} \right] \\ &= \gamma + 2\omega_{ct}^{sb} - \omega_{uc}^{ds} .\end{aligned}\tag{20}$$

We see that this phase is γ plus angles in the sb and ds triangles. From Eq. (18), we note that the particular sb and ds angles involved are the small ones, so that $\varphi_{s,D_s^+ K^-}$ is γ plus a $\leq O(\lambda^2)$ correction.

In the same way, we can find the CKM phases probed by the other $B_s(t)$ decay modes listed in Table 1. For the decays

$$B^\pm \rightarrow D + K^\pm ,\tag{21}$$

f_{CP}

where f_{CP} is the CP eigenstate (e.g. K^+K^-) into which the neutral D decays, we must find the relative CKM phase of the two interfering terms in the decay amplitude

$$\begin{aligned}A \left(\begin{matrix} B^+ \rightarrow D + K^+ \\ f_{CP} \end{matrix} \right) &= A(B^+ \rightarrow D^0 K^+) A(D^0 \rightarrow f_{CP}) \\ &\quad + A(B^+ \rightarrow \overline{D}^0 K^+) A(\overline{D}^0 \rightarrow f_{CP}).\end{aligned}\tag{22}$$

If $D^0\text{-}\overline{D}^0$ mixing is slow compared to the D^0 decay rate, then, as suggested by Eq. (22), the D -system phases which influence the decay sequence (21) are the D decay phases, not the $D^0\text{-}\overline{D}^0$ mixing phase. But then the phase probed by the sequence depends on f_{CP} . For $f_{CP} = K^+K^-$, we find from the diagrams of Figs. 7 and 8, and the tree diagrams for $D^0 \rightarrow K^+K^-$ and $\overline{D}^0 \rightarrow K^+K^-$, that the relative CKM phase of the two terms in Eq. (22) is

$$\begin{aligned}\arg[V_{us}V_{ub}^*/V_{cs}V_{cb}^*] &= \omega_{uc}^{sb} \\ &= \arg \left[\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \frac{V_{us}V_{ud}^*}{V_{cs}V_{cd}^*} \right] = \gamma - \omega_{uc}^{ds} .\end{aligned}\tag{23}$$

Thus, the decay chain (21) with $f_{CP} = K^+K^-$ probes a CKM phase which is one of the “large” angles in the sb triangle, and this angle is in turn γ plus a $\leq O(\lambda^4)$ correction.

In Table 2 we show what CKM phases are actually probed by the various “ γ modes” we have considered when the CKM matrix is treated exactly. These phases are expressed in terms of γ and angles in the sb and ds triangles. We see from Table 2 that *none* of the modes we have discussed actually measures γ . Every one of them yields γ , or 2γ , plus nonzero

Decay Mode	CKM Phase Probed
$B_s, \bar{B}_s \rightarrow D_s^\pm K^\mp$	$\gamma + 2\omega_{ct}^{sb} - \omega_{uc}^{ds}$
$B_s, \bar{B}_s \rightarrow D^0 \phi, \bar{D}^0 \phi$	$\gamma + 2\omega_{ct}^{sb} - \omega_{uc}^{ds}$
$B_s, \bar{B}_s \rightarrow \rho^0 K_S$	$2(\gamma + \omega_{ct}^{sb})$
$B^\pm \rightarrow DK^\pm$ [with $D \rightarrow K^+ K^-$]	$\gamma - \omega_{uc}^{ds}$
$B_d(\bar{B}_d) \rightarrow DK^{*0}(D\bar{K}^{*0})$ [with $D \rightarrow K^+ K^-$]	$\gamma - \omega_{uc}^{ds}$

Table 2: CKM phases probed by the “ γ modes” when the CKM matrix is treated exactly.

corrections. On the other hand, in every case the corrections involve only the $\leq O(\lambda^2)$ angle in the sb triangle and/or the $\leq O(\lambda^4)$ angle in the ds triangle. Thus, the corrections are always less than 0.1 radians. One might wonder whether the correction angles ω_{ct}^{sb} and ω_{uc}^{ds} can represent a *fractionally* large correction in the event that γ itself is small. It can be shown that they cannot. When γ goes to zero, ω_{ct}^{sb} and ω_{uc}^{ds} also go to zero, at the same rate as γ . Furthermore, given what is already known about the CKM matrix, the proportionality constant relating ω_{ct}^{sb} to γ for small γ is ~ 0.015 , and that relating ω_{uc}^{ds} to γ is still smaller. Thus, the corrections to γ are always fractionally small.

As the examples in Table 2 suggest, *any* B decay mode which probes γ in the Wolfenstein approximation probes γ plus, at most, corrections involving only the small angles ω_{ct}^{sb} and ω_{uc}^{ds} when the CKM matrix is treated exactly. For, as discussed in Sec. 4, the exact CKM phase probed by an arbitrary B decay mode can be expressed as a linear combination of γ , β , ω_{ct}^{sb} and ω_{uc}^{ds} , with integer coefficients. Now, for a B decay which yields γ in the Wolfenstein approximation, this linear combination obviously does not involve β . Thus, apart from γ , it can involve only ω_{ct}^{sb} and ω_{uc}^{ds} .

While the angles measured by the various “ γ modes” differ only slightly from γ , they do differ, and in some modes they may differ by as much as 0.05 to 0.1. In contrast, neglecting penguin contributions, the “ α modes” $B_d, \bar{B}_d \rightarrow \pi^+ \pi^-$ and $B_d, \bar{B}_d \rightarrow \rho^\pm \pi^\mp$ yield precisely α , even when the CKM matrix is treated exactly. Similarly, assuming as usual that K^0 - \bar{K}^0 mixing is dominated by the $d\bar{s} \rightarrow c\bar{c} \rightarrow s\bar{d}$ box diagram, the “ β mode” $B_d, \bar{B}_d \rightarrow \Psi K_S$ yields precisely β . Thus, in the second generation experiments on CP violation in B decays, it would be interesting to test the Standard Model by showing that the angles extracted from, say, the modes $B_d, \bar{B}_d \rightarrow \pi^+ \pi^-$, $B_d, \bar{B}_d \rightarrow \Psi K_S$ and $B_s, \bar{B}_s \rightarrow D_s^\pm K^\mp$ *fail* to add up to π by an amount of order 0.05 to 0.1. To carry out this precision test of the angles probed by the leading diagrams in various modes, one would have to eliminate from $B_d, \bar{B}_d \rightarrow \pi^+ \pi^-$ the possible penguin contributions, using an isospin analysis.⁷

It is tempting to ask whether there is *any* B decay mode which, unlike all the modes we have discussed, actually measures precisely γ when the CKM matrix is not approximated. In principle, the mode $B_c^+ \rightarrow D^0 \pi^+$ does this via interference between the diagrams shown in Fig. 10. The relative CKM phase of these diagrams is just $\arg(V_{ud}V_{ub}^*/V_{cd}V_{cb}^*) = \gamma$. However, it is not clear that the penguin diagrams for this decay are small relative to the annihilation diagram in Fig. 10,¹⁵ and, in any case, this mode, like most charged B decays, cannot yield clean CKM phase information.

It would, of course, be very interesting to probe directly angles in the sb and ds triangles. One decay mode that would do this is $B_s(t), \bar{B}_s(t) \rightarrow \Psi \phi$.¹⁶ In this mode, the

Figure 10: Diagrams for $B_c^+ \rightarrow D^0 \pi^+$.

interfering decay amplitudes are $A(B_s \rightarrow \Psi\phi)$, which is dominated by the diagram in Fig. 11, and $A(\overline{B}_s \rightarrow \Psi\phi)$, which is dominated by its CP conjugate. From Eqs. (10) and (4), the CKM phase probed by $B_s(t) \rightarrow \Psi\phi$ is then¹⁷

$$\begin{aligned}\varphi_{s,\Psi\phi} &= \arg \left[\frac{V_{cs}V_{cb}^*}{\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} V_{cb}V_{cs}^*} \right] \\ &= 2 \arg \left[\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*} \right] = 2\omega_{ct}^{sb},\end{aligned}\tag{24}$$

just twice the small angle in the sb triangle.

To use this mode, tagging and measurement of the time dependence will, of course, be necessary. The final state $\Psi\phi$ is technically not a CP eigenstate, because it may be a mixture of helicity configurations. However, it appears that the outgoing particles in the decay $B_d \rightarrow \Psi K^*$ have zero helicity much of the time.¹⁸ We then expect the same to be true of the outgoing particles in $B_s(t), \overline{B}_s(t) \rightarrow \Psi\phi$. The final state then *is* largely a CP eigenstate, and the decay rates are described by the simple equations (6).

By comparing to the very similar decay $B_d \rightarrow \Psi K^*$, one readily estimates that $BR(B_s \rightarrow \Psi\phi) \simeq 10^{-3}$, a very promising value.

It has been pointed out¹⁶ that $\sin \varphi_{s,\Psi\phi}$, the quantity probed by $B_s(t), \overline{B}_s(t) \rightarrow \Psi\phi$ via Eqs. (6), can be rewritten as

$$|\sin \varphi_{s,\Psi\phi}| = 2 \left| V_{cd} \frac{V_{ub}}{V_{cb}} \sin \gamma \right| (1 + O(\lambda^2)).\tag{25}$$

Figure 11: The dominant diagram for $B_s \rightarrow \Psi\phi$.

Thus, if $|V_{ub}/V_{cb}|$ is known, this decay mode becomes another way to determine γ . Of course, the mode does not “probe γ ” in the sense of involving two interfering amplitudes whose relative CKM phase is γ or 2γ . Rather, the relative phase is, as we saw in Eq. (24), the small phase $2\omega_{ct}^{sb}$. The CP-violating asymmetry in the mode will be correspondingly small, rather than being of order $\sin\gamma$ or $\sin 2\gamma$. Indeed, if we assume that $|V_{ub}/V_{cb}| \sim 0.07$,¹³ then Eqs. (25) and (6) indicate that the asymmetry will be $\sim 0.03 \sin\gamma$, which is necessarily quite small. Nevertheless, perhaps the large branching ratio for the mode will make this small asymmetry observable. Needless to add, if $B_s(t), \bar{B}_s(t) \rightarrow \Psi\phi$ should be found to have an asymmetry much larger than, say, 0.06, then we would have evidence for a CP-violating mechanism beyond that in the Standard Model.

4. THE UNITARITY TRIANGLES AND THE CKM MATRIX

The discussion of the previous Section calls attention to the unitarity triangles beyond the db triangle, and to the angles in those other triangles. We would now like to briefly report the results of a general analysis of how the angles in the full set of six unitarity triangles are related to CP-violating asymmetries in B decay, and of how they are related to the CKM matrix. A more complete discussion, including the proofs of the results, will be presented elsewhere.¹⁹

There are three main results, which we shall describe in turn.

- As we have already noted (see Eq. 17), each angle in a “row” triangle is also an angle in a “column” triangle. Thus, there are at most nine, not eighteen, distinct angles in the six unitarity triangles. We find that exactly four of these nine angles are independent. The remaining five angles are very simple linear combinations of the independent four. The independent angles may be chosen, for example, as two of the angles α , β and γ in the db triangle, plus two of the angles in one of the other column triangles. They may also be chosen as two of the angles α , β and γ , plus the two small angles ω_{ct}^{sb} and ω_{uc}^{ds} .
- As we have seen, any CP-violating asymmetry in B decay probes the relative CKM phase of two interfering amplitudes. Thus, the asymmetry probes the phase of some product and quotient, or, equivalently, of some product, of CKM elements. Now, not every conceivable product of CKM elements has a phase which is invariant under phase redefinitions of the quark fields. However, if the phase of some product of CKM elements is determined by an experiment, then, obviously, this phase must be invariant under quark-field rephasing.

We find that if the phase φ of some product of CKM elements is rephasing-invariant, then

$$\varphi = \sum_{i=1}^4 n_i \varphi_i + k\pi, \quad (26)$$

where the φ_i are the four independent angles in the unitarity triangles, the n_i are integers, and k is zero or one. For any specific φ and choice of the independent angles φ_i , the n_i and k will, of course, be known quantities.

The relation (26) states that the CKM phase probed by any CP-violating asymmetry in a B decay is a simple linear combination, with integer coefficients, of the four independent angles in the unitarity triangles. Thus, these angles form a complete set of variables for the description of CP violation in B decay. Moreover, these variables are very closely and simply related to the quantities – the phases φ – to be measured in B decay experiments.

- Suppose that the four independent angles φ_i have been determined by CP-violation experiments. What have we learned about the CKM matrix? The answer is that once the φ_i are known, the entire CKM matrix follows from them! That is, in principle at least, we can determine the whole CKM matrix, including the magnitudes of all its elements, and all its physically-meaningful phases, through measurements of nothing but CP-violating asymmetries in B decays. Thus, albeit at varying levels of sensitivity, these CP-violating asymmetries probe *everything* in the CKM matrix. Consequently, they are potentially a very rich test of the Standard Model explanation of CP violation.

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